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ABSTRACT

J. Barnette and J. McLean (1996) proposed a method of controlling Type I error in pairwise multiple comparisons after a significant omnibus F test. This procedure, called Alpha-Max, is based on a sequential cumulative probability accounting procedure in line with Bonferroni inequality. A missing element in the discussion of Alpha-Max was the empirical determination of actual probabilities of Type I errors. This paper compares the Type I error rates of Alpha-Max with other commonly used multiple comparison procedures: (1) Fisher's Least Significant Difference (LSD); (2) Dunn-Bonferroni; (3) Tukey's Honestly Significant Difference (HSD); (4) the Student Newman Keuls (SNK) procedure; and (5) the Scheffe approach. Monte Carlo procedures were used to generate 10,000 replications with varied alpha of 0.05 and 0.01; 3, 4, and 5 groups; and 5 sample sizes. Actual Type I error rates were determined for the greatest difference and for total number of Type I errors. Results indicate that in virtually every situation LSD and Alpha-Max had significantly higher probability of Type I errors than the other four methods. SNK and HSD had higher than nominal alpha probabilities for committing Type I errors, with SNK having a lower level than HSD. Dunn-Bonferroni had a level slightly lower than the nominal level, while the Scheffe had a level much lower than the nominal level. Varying sample size had little practical significance. While Alpha-Max did not provide for acceptable experiment-wise control of Type I error, it may provide an alternative for control of Type I error in the planned, nonorthogonal situation or in situations where assumptions of analysis of variance are violated. (Contains 25 tables and 4 references.) (Author/SLD)

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A Comparison of Type I Error Rates of Alpha-Max with Established Multiple Comparison Procedures

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ABSRACT

Barnette and McLean (1996, November) proposed a method of controlling Type I error in pairwise multiple comparisons after a significant omnibus F test. This procedure, called Alpha-Max, was based on a sequential cumulative probability accounting procedure in line with the Bonferroni inequality. A missing element in the discussion of Alpha-Max was the empirical determination of actual probabilities of Type I errors. This paper compares the Type I error rates of Alpha-Max with five other commonly used multiple comparison procedures: Fisher's LSD, Dunn-Bonferroni, Tukey's HSD, SNK, and the Scheffe' approach, as applied to pairwise differences. Monte Carlo procedures were used to generate 10,000 replications, sampling from a unit normal population, in every combination of three factors: alpha of .05 and .01; number of groups of 3, 4, and 5; and sample sizes of 5, 20, 35, and 50. Actual Type I error rates were determined for the greatest difference and for total number of Type I errors. These were compared using a two-way design of the multiple comparison procedure crossed with sample sizes. This analysis was conducted within each combination of alpha and number of groups.

Results indicate that in virtually every situation LSD and Alpha-Max had significantly higher probability of Type I errors than the other four methods. SNK and HSD had higher than nominal alpha probabilities of committing Type I errors, with SNK having a higher level than HSD. Dunn-Bonferroni had a level slightly lower than the nominal level while the Scheffé had a level much lower than the nominal level. Varying sample size had little practical significance. While Alpha-Max did not provide for acceptable experimentwise control of Type I error, it may provide an alternative for control of Type I error in the planned, non-orthogonal situation or in situations where assumptions of ANOVA are violated.

A Comparison of Type I Error Rates of Alpha-Max with Established Multiple Comparison Procedures

A novel and potentially useful new multiple comparison procedure (MCP) was introduced in 1996 by Barnette and McLean (1996, November). They referred to this new procedure as Alpha-Max. They used selective data to demonstrate Alpha-Max's apparent superiority for controlling Type I error rates to Fisher's Least Significant Difference (LSD) procedure with the Bonferroni correction, Tukey's Honestly Significant Difference (HSD) procedure, and the Student Newman Keuls (SNK) procedure. However, this was done with data sets purposefully and selectively configured to model specific situations. The purpose of the present research is to compare empirically determined Type I error rates of Alpha-Max, Fisher's LSD procedure, the SNK procedure, Tukey's HSD procedure, a pairwise application of the Dunn (Bonferroni) procedure, and a pairwise application of the Scheffé procedure using Monte Carlo procedures.

It should be noted that this study is limited to consideration of Type I errors. The authors recognize the importance of power, effect size, and replication, but since the primary purpose of this study is the evaluation of the relatively new Alpha-Max procedure, it was limited to the first criteria considered by most researchers, the probability of making Type I errors.

Background

Multiple comparison procedures were developed as followups to an omnibus F-test in the analysis of variance (ANOVA). It quickly became apparent that the Type I error rates would be

inflated if multiple significance tests without some control were used. Type I error rates can be described for a number of situations. One situation is where Type I error is described to cover a specified number of tests simultaneously. For example, an hypothesis-wise error rate refers to the chance of making a Type I error for one hypothesis or, in this case, one comparison. On the other hand, an experiment-wise Type I error rate refers to the chance of making an error for all of the tests in a complete experiment or at least for a family of comparisons. In this paper, we refer to hypothesis-wise error rate as the probability of making a Type I error for one test of a hypothesis and an experiment-wise error rate as the probability of making at least one Type I error when testing all of the pairwise comparisons following a significant omnibus F-test in ANOVA.

Traditional comparison of the procedures' control of Type I errors, it has been done traditionally using two approaches. The first is the comparison of only the largest difference. While this approach does provide assistance to a research or trying to identify which MCP to use, the purpose for using an MCP is to identify all differences. Thus, a more useful approach would be to compare the various MCPs in their abilities to control error for identifying all pairwise differences.

As previously noted, the present study will compare Alpha-Max, Fisher's LSD procedure, the SNK procedure, Tukey's HSD procedure, a pairwise application of the Dunn (Bonferroni) procedure, and a pairwise application of the Scheffé procedure

using Type I error rates as the dependent variable. Each of these procedures is described briefly below:

Alpha-Max. The Alpha-Max procedure uses the a priori alpha (Type I probability) by ordering the pair-wise difference p-values from lowest to highest, accumulating the actual p-values from lowest to highest until the addition of the next highest p-value exceeds the pre-set nominal alpha (α) level. All pair-wise differences whose p-values are already included in the set are considered significant, but the one(s) that result(s) in the sum of the p-values being higher than the a priori α is/are not considered significant (Barnette & McLean, 1996, November).

Fisher's LSD Procedure. The LSD procedure is equivalent to conducting all pair-wise comparisons using independent t-tests with the MS_{error} as the common pooled variance estimate (Kirk, 1982).

SNK Procedure. The SNK uses Student's t-statistic in a layered approach. First the differences between means are ordered from largest to smallest differences. The pairs are grouped based on their location in the ordered set. Those at each extreme are in the first group, those next extreme in the second group, and so on until the last group has those adjacent to each other in magnitude. A critical value is computed for each group using the HSD procedure with the number of groups set to the number of means between the ordered pairs including the two means being compared (Kirk, 1982).

Dunn (Bonferroni) Procedure. The Dunn procedure uses the Bonferroni inequality as authority to divide equally the a priori

error among the number of tests to be completed. For example, if the a priori α is .05 and 10 tests are required, each test will be run at the .005 level (Hayes, 1988).

Scheffé Procedure. The Scheffé procedure allows for all possible comparisons by adjusting the critical F-value. The adjustment is to use the F-value for the omnibus F-test multiplied by the omnibus degrees of freedom as the critical F-value for each comparison (Kirk, 1982).

The *Dictionary of Statistics and Methodology* defines Monte Carlo Methods as "any generating of random values (most often with a computer) to study statistical models" (Vogt, 1993, p. 143). Probably its most well-known application was in the classic study of ANOVA and ANCOVA assumptions published in the *Review of Educational Research* in 1972 (Glass, Peckham, & Sanders). Monte Carlo techniques combined with the power of modern computers has opened a new world of research possibilities. Situations that are too complex or cumbersome for analytic solutions can be explored empirically using Monte Carlo methods. Statistical modeling based on Monte Carlo procedures has become common in disciplines ranging from the hard sciences to the social sciences. Who cannot relate to the weather forecasts that often use computer modeling techniques? Monte Carlo methods have become a recognized tool in educational research.

Research Questions

The following research questions are addressed in this study:

1. When α is set at .05, how do the proportions of identifying at least one Type I error compare among the six multiple comparison procedures when K= 3, K= 4, and K= 5?

2. When α is set at .01, how do the proportions of identifying at least one Type I error compare among the six multiple comparison procedures when K= 3, K= 4, and K= 5?

3. When α is set at .05, how do the proportions of total Type I errors compare among the six multiple comparison procedures when K= 3, K= 4, and K= 5?

4. When α is set at .01, how do the proportions of total Type I errors compare among the six multiple comparison procedures when K= 3, K= 4, and K= 5?

Procedures

The basic design used was a procedures (6 levels) by sample size (4 levels) factorial design with 10 observations per cell, repeated and analyzed separately for six combinations of number of groups (3, 4, and 5) and two levels of α (.05 and .01). For each situation, there were six multiple comparison procedures used to analyze the resulting data sets: LSD, Alpha-Max, Student-Newman-Keuls, HSD, Dunn, and Scheffé crossed with four sample sizes: 5, 20, 35, and 50 per sample.

There were two dependent variables: proportion of times there was at least one Type I error (significant difference found for largest pairwise mean difference) and total proportion of Type I errors (total number of significant differences).

Each cell had ten observations. These observations were determined in the following way:

1. Sample size (n), number of groups (K), and α level were set.

2. Observations (n) were randomly generated from a normal distribution of z score deviates for each of the K groups.

3. Means were computed for each group.

4. These means were compared using one-way ANOVA.

5. If the omnibus F statistic was significant at the level of significance, pairwise differences were compared using each of the six multiple comparison procedures.

6. When the multiple comparison procedures were used, two observations were recorded for each procedure: determination of at least one significant difference and total number of significant differences.

7. Steps 1 through 7 were repeated 1000 times.

8. Number of times per 1000 at least one significant difference was found (for largest pairwise mean difference) and total number of significant differences were recorded for each multiple comparison procedure; these were converted to proportions which became the unit of analysis.

9. Steps 1 through 8 were repeated 10 times using a PC program written by the first author (PMMCP) in double-precision QBASIC, compiled and run on a Pentium 90, Gateway PC.

10. Data generated by PMMCP were saved as PC data files which were then transferred to a mainframe computer for final analysis using SAS®.

11. Steps 1 through 10 were repeated for each group number (K= 3, K= 4, and K= 5) crossed with each nominal α (.05 and .01).

For each of the six groups by α combination a two-way ANOVA was conducted with multiple comparison procedure crossed with sample size. Three effects were examined: the interaction of MCP and sample size, and, if not significant, main effect tests were conducted for the MCP and sample size independent variables. In addition, practical significance was reported using η^2 (SS_{effect}/SS_{total}).

Results

Results are presented in four sections, each relating to one of the research questions. A brief discussion of the results is also presented in the four sections.

At least one Type I error when $\alpha = .05$

Research question 1 is:

1. When α is set at .05, how do the proportions of identifying at least one Type I error compare among the six multiple comparison procedures when K= 3, K= 4, and K= 5?

The focus here is on finding at least one significant difference related to the largest pairwise mean difference. Of course, in this case the MCP's are not unique. The LSD and Alpha-Max will be the same since, if the largest difference has an actual probability less than α, LSD will identify the difference as being significant and so will Alpha-Max since the accumulated α will also be less than α. Also, since the SNK and HSD are the same at the highest number of steps, also the largest difference, they will be consistent. Thus, the primary differences here are among the LSD/Alpha-Max set, the SNK/HSD set, the Dunn, and the Scheffé.

Results related to this question are presented in Tables 1 through 6. Table 1 presents the proportion of Type I errors found with the largest pairwise mean difference by MCP and sample size for three groups, and Table 2 presents the results of the two-way ANOVA. There was no significant interaction of MCP and sample size. There was a significant main effect for sample size difference; however, the practical significance was low (.103). There was a significant main effect among the MCP's with a

relatively high practical significance (.235). The LSD/Alpha-Max MCP had a proportion of .0524, slightly higher than α; while the SNK/HSD had a proportion of .0474, slightly lower than α; the Dunn MCP had a proportion of .0433; and the Scheffé procedure had a proportion of .0407. Follow-up, using the HSD procedure, indicated the LSD/Alpha-Max set had a significantly higher proportion than any of the other MCP's; the SNK/HSD set had a higher proportion than the Scheffé but not higher than the Dunn. In the K= 3 situation, LSD and Alpha-Max were liberal, while the other methods were conservative as compared with the nominal α level.

Table 3 presents the proportion of Type I errors found with the largest pairwise mean difference by MCP and sample size for four groups, and Table 4 presents the results of the two-way ANOVA. There was no significant interaction of MCP and sample size. There was a significant main effect for sample size difference; however, the practical significance was low (.071). There was a significant main effect among the MCP's with a very high practical significance (.545). The LSD/Alpha-Max MCP had a proportion of .0476, slightly lower than α, while the SNK/HSD had a proportion of .0402, lower than α; the Dunn MCP had a proportion of .0349; and the Scheffé procedure had a proportion of .0256, both much lower than α. Follow-up, using the HSD procedure, indicated the LSD/Alpha-Max set had a significantly higher proportion than any of the other MCP's, the SNK/HSD set had a higher proportion than the Dunn and Scheffé, and the Dunn had a higher proportion than the Scheffé. In the K= 4 situation,

all six of the methods were conservative as compared with the nominal α level.

Table 5 presents the proportion of Type I errors found with the largest pairwise mean difference by MCP and sample size for five groups, and Table 6 presents the results of the two-way ANOVA. There was no significant interaction of MCP and sample size. There was a significant main effect for sample size difference; however, the practical significance was low (.010). There was a significant main effect among the MCP's with a very high practical significance (.779). The LSD/Alpha-Max MCP had a proportion of .0499, essentially at α, while the SNK/HSD had a proportion of .0403, lower than α; the Dunn MCP had a proportion of .0347; and the Scheffé procedure had a proportion of .0190, both much lower than α. Follow-up, using the HSD procedure, indicated the LSD/Alpha-Max set had a significantly higher proportion than any of the other MCP's, the SNK/HSD set had a higher proportion than the Dunn and Scheffé, and the Dunn had a higher proportion than the Scheffé. In the K= 5 situation, LSD and Alpha-Max were at the nominal α level, while the other four methods were conservative as compared with the nominal α level.

In general, all six of the methods provided good control of Type I error for at least one pairwise difference, particularly when K was greater than three. LSD, Alpha-Max, SNK, and HSD provided proportions relatively close to α, while the Dunn and Scheffé approaches were very conservative. There was no interaction of MCP and sample size, and while there was a

significant main effect for sample size, this was not accompanied by practical significance.

At least one Type I error when $\alpha = .01$

Research question 2 is:

2. When α is set at .01, how do the proportions of identifying at least one Type I error compare among the six multiple comparison procedures when K= 3, K= 4, and K= 5?

Again, the focus here is on finding at least one significant difference related to the largest pairwise mean difference. Of course, in this case the MCP's are not unique. The LSD and Alpha-Max will be the same, since if the largest difference has an actual probability less than α, LSD will identify the difference as being significant and so will Alpha-Max because the accumulated α will also be less than α. Also, since the SNK and HSD are the same at the highest number of steps, also the largest difference, they will be consistent. Thus, the primary differences here are among the LSD/Alpha-Max set, the SNK/HSD set, the Dunn, and the Scheffé.

Results related to this question are presented in Tables 7 through 12. Table 7 presents the proportion of Type I errors found with the largest pairwise mean difference by MCP and sample size for three groups, and Table 8 presents the results of the two-way ANOVA. There was no significant interaction of MCP and sample size. There was a significant main effect for sample size difference; however, the practical significance was low (.037). There was a significant main effect among the MCP's with a relatively low practical significance (.089). The LSD/Alpha-Max MCP had a proportion of .0103, slightly higher than α; while the

SNK/HSD had a proportion of .0093; slightly lower than α, the Dunn MCP had a proportion of .0900, and the Scheffé procedure had a proportion of .0079. Follow-up, using the HSD procedure, indicated the LSD/Alpha-Max set had a significantly higher proportion than the Scheffé. In the K= 3 situation, LSD, Alpha-Max, SNK, HSD, and Dunn were relatively close to the nominal α, while the Scheffé was conservative as compared with the nominal α level.

Table 9 presents the proportion of Type I errors found with the largest pairwise mean difference by MCP and sample size for four groups, and Table 10 presents the results of the two-way ANOVA. There was no significant interaction of MCP and sample size. There was a significant main effect for sample size difference; however, the practical significance was low (.068). There was a significant main effect among the MCP's with a very high practical significance (.449). The LSD/Alpha-Max MCP had a proportion of .0103, very close to α, while the SNK/HSD had a proportion of .0081, slightly lower than α; the Dunn MCP had a proportion of .0072; and the Scheffé procedure had a proportion of .0045, both much lower than α. Follow-up, using the HSD procedure, indicated the LSD/Alpha-Max set had a significantly higher proportion than any of the other MCP's, the SNK/HSD set had a higher proportion than the Scheffé, and the Dunn had a higher proportion than the Scheffé. In the K= 4 situation, the LSD/Alpha Max procedures were close to the nominal α while the other four procedures were conservative, the Scheffé being very conservative.

Table 11 presents the proportion of Type I errors found with the largest pairwise mean difference by MCP and sample size for five groups, and Table 12 presents the results of the two-way ANOVA. There was no significant interaction of MCP and sample size, nor was there a significant main effect for sample size difference. There was a significant main effect among the MCP's with a very high practical significance (.458). The LSD/Alpha-Max MCP had a proportion of .0104, essentially at α, while the SNK/HSD had a proportion of .0078, lower than α; the Dunn MCP had a proportion of .0068; and the Scheffé procedure had a proportion of .0031, both much lower than α. Follow-up, using the HSD procedure, indicated the LSD/Alpha-Max set had a significantly higher proportion than any of the other MCP's, the SNK/HSD set had a higher proportion than the Scheffé, and the Dunn had a higher proportion than the Scheffé. In the K= 5 situation, LSD and Alpha-Max were very close to the nominal α level, while the other four methods were conservative as compared with the nominal α level. The Scheffé procedure was very conservative.

In general, all six of the methods provided good control of Type I error for at least one pairwise difference. LSD and Alpha-Max provided proportions very close to α, SNK, and HSD provided proportions lower than α, while the Dunn and Scheffé approaches were very conservative. There was no interaction of MCP and sample size, and while there was a significant main effect for sample size, this was not accompanied by practical significance.

If the concern is control of Type I error for having at least one in a given experiment, then all six of these procedures are reasonable approaches. LSD and Alpha-Max provide for control very close to nominal levels, SNK and HSD tend to be slightly conservative, Dunn is very conservative, and Scheffé is extremely conservative. However, a true experimentwise Type I error control needs to be sensitive to the proportion of total number of Type I errors that will occur using these six procedures. The next sections of this paper deal with this issue.

Total proportion of Type I errors when $\alpha = .05$

Research question 3 is:

3. When α is set at .05, how do the proportions of total Type I errors compare among the six multiple comparison procedures when K= 3, K= 4, and K= 5?

The focus here is the total number of Type I errors. In this case the MCP's are unique as discussed earlier in this paper. In general, it is expected, based on the theoretical foundations of these procedures, the order of MCP's on a scale of liberal to conservative is: LSD, Alpha-Max, SNK, HSD, Dunn, and Scheffé.

Results related to this question are presented in Tables 13 through 18. Table 13 presents the proportion of total number of Type I errors by MCP and sample size for three groups, and Table 14 presents the results of the two-way ANOVA. There was no significant interaction of MCP and sample size. There was a significant main effect for sample size difference; however, the practical significance was relatively low (.121). There was a significant main effect among the MCP's with very high practical

significance (.544). The LSD MCP had a proportion of .0813, the Alpha-Max had a proportion of .0734, the SNK had a proportion of .0720, and the Dunn had a proportion of .0573, all higher than α. The Dunn had a proportion of .0514, slightly higher than α; and the Scheffé procedure had a proportion of .0478; slightly lower than α. Follow-up, using the HSD procedure, indicated the LSD had a significantly higher proportion than the other five MCP's. Alpha-Max and SNK were significantly higher than HSD, Dunn, and Scheffé. HSD was significantly higher than Scheffé. In the K= 3 situation, the Dunn MCP resulted in a Type I error rate closest to the nominal level.

Table 15 presents the proportion of total number of Type I errors found by MCP and sample size for four groups, and Table 16 presents the results of the two-way ANOVA. There was no significant interaction of MCP and sample size. There was a significant main effect for sample size difference; however, the practical significance was low (.016). There was a significant main effect among the MCP's with a very high practical significance (.829). The LSD had a proportion of .1086, and Alpha-Max had a proportion of .0950, both very liberal. The SNK had a proportion of .0693, also liberal. The HSD had a proportion of .0515, slightly higher than α; the Dunn MCP had a proportion of .0434, slightly lower than α; and the Scheffé procedure had a proportion of .0302, much lower than α. Follow-up, using the HSD procedure, indicated the LSD had a significantly higher proportion than all of the other MCPs; Alpha-Max was higher than SNK, HSD, Dunn, and Scheffé. SNK was

higher than HSD, Dunn, and Scheffé. HSD was higher than Dunn and Scheffé, and Dunn was higher than Scheffé. In the K= 4 situation, the Tukey's HSD had an observed Type I error closest to α. LSD, Alpha-Max, and SNK were liberal, while Dunn and Scheffé were conservative.

Table 17 presents the proportion of total number of Type I errors found by MCP and sample size for five groups, and Table 19 presents the results of the two-way ANOVA. There was no significant interaction of MCP and sample size. There was a significant main effect for sample size difference, however the practical significance was very low (.006). There was a significant main effect among the MCP's with an extremely high practical significance (.917). The LSD MCP had a proportion of .1627 and the Alpha-Max had a proportion of .1310, both very liberal. The SNK had a proportion of .0790, liberal, while the HSD was less liberal with a proportion of .0577. The Dunn MCP had a proportion of .0466, slightly conservative, and the Scheffé procedure had a proportion of .0236, very conservative. Follow-up, using the HSD procedure, indicated the LSD had a significantly higher proportion than all of the other MCPs; Alpha-Max was higher than SNK, HSD, Dunn, and Scheffé. SNK was higher than HSD, Dunn, and Scheffé. HSD was higher than Dunn and Scheffé, and Dunn was higher than Scheffé. In the K= 5 situation, the Dunn had an observed Type I error closest to α. LSD, Alpha-Max, and SNK were very liberal, HSD was somewhat liberal, while the Scheffé was very conservative.

Total proportion of Type I errors when $\alpha = .01$

Research question 4 is:

4. When α is set at .01, how do the proportions of total Type I errors compare among the six multiple comparison procedures when K= 3, K= 4, and K= 5?

Again, the focus here is the total number of Type I errors. In general, it is expected, based on the theoretical foundations of these procedures, the order of MCP's on a scale of liberal to conservative is: LSD, Alpha-Max, SNK, HSD, Dunn, and Scheffé.

Results related to this question are presented in Tables 19 through 24. Table 19 presents the proportion of total number of Type I errors by MCP and sample size for three groups, and Table 20 presents the results of the two-way ANOVA. There was no significant interaction of MCP and sample size. There was a significant main effect for sample size difference; however, the practical significance was relatively low (.041). There was a significant main effect among the MCP's with relatively high practical significance (.216). The LSD MCP had a proportion of .0138, the Alpha-Max had a proportion of .0129, the SNK had a proportion of .0121. The HSD had a proportion of .0105, slightly higher than α. The Dunn had a proportion of .0101, very close to the nominal α, and the Scheffé procedure had a proportion of .0088, slightly lower than α. Follow-up, using the HSD procedure, indicated the LSD and Alpha-Max had significantly higher proportions than HSD, Dunn, and Scheffé, but not higher than SNK. SNK was significantly higher than Scheffé. In the K= 3 situation, the Dunn MCP resulted in a Type I error rate closest to the nominal level. However, HSD was very close.

Table 21 presents the proportion of total number of Type I errors found by MCP and sample size for four groups, and Table 22 presents the results of the two-way ANOVA. There was no significant interaction of MCP and sample size. There was a significant main effect for sample size difference; however, the practical significance was low (.037). There was a significant main effect among the MCP's with a very high practical significance (.645). The LSD had a proportion of .0190, and Alpha-Max had a proportion of .0174, both much higher than α. The SNK had a proportion of .0113, slightly higher than α. The HSD had a proportion of .0098, very close to α. The Dunn MCP had a proportion of .0083, slightly lower than α, and the Scheffé procedure had a proportion of .0049, much lower than α. Follow-up, using the HSD procedure, indicated the LSD and Alpha-Max had significantly higher proportions than SNK, HSD, Dunn, and Scheffé. SNK was higher than Dunn and Scheffé. HSD and Dunn were higher than Scheffé. In the K= 4 situation, the Tukey's HSD had an observed Type I error closest to α. LSD, Alpha-Max, and SNK were liberal, while Dunn and Scheffé were conservative.

Table 23 presents the proportion of total number of Type I errors found by MCP and sample size for five groups, and Table 24 presents the results of the two-way ANOVA. There was no significant interaction of MCP and sample size, nor was there a significant main effect for sample size difference. There was a significant main effect among the MCP's with a very high practical significance (.626). The LSD MCP had a proportion of .0261 and the Alpha-Max had a proportion of .0224, both very

liberal. The SNK had a proportion of .0126, somewhat higher than α. The HSD had a proportion of .0103, very close to α. The Dunn MCP had a proportion of .0086, slightly conservative, and the Scheffé procedure had a proportion of .0036, very conservative. Follow-up, using the HSD procedure, indicated the LSD and Alpha-Max had significantly higher proportions than SNK, HSD, Dunn, and Scheffé. SNK was higher than Dunn and Scheffé. HSD and Dunn were higher than Scheffé. In the K= 5 situation, the Tukey's HSD had an observed Type I error closest to α. LSD, Alpha-Max, and SNK were liberal, while Dunn and Scheffé were conservative.

Summary of Total Type I Error Proportions

Table 25 summarizes the observed Type I error rates for the six MCP's for the combinations of group size and α level. These results indicate that only two of the procedures provide relatively accurate control of Type I error on an experimentwise basis: Tukey's HSD and the Dunn procedure. Of these, the Tukey tends to be a little liberal and the Dunn tends to be a little conservative. Clearly, LSD and Alpha-Max do not provide reasonable control of Type I error; and SNK is less liberal compared with these, but still has a higher Type I error rate than the nominal α.

It is interesting to note that total Type I error rates for LSD and Alpha-Max increase with an increase in the number of groups. Of course, with more groups, there are more pairs which may possibly be significant. It would be interesting to determine whether this increase is a function of the number of pairwise comparisons or a function of the procedures themselves.

The Scheffé Type I error rate becomes more conservative as the number of groups increases.

These results indicate that Alpha-Max does not appear to be a viable procedure to use for pairwise follow-up. It may, however, be an alternative to use for planned comparisons.

Summary and Recommendations

When the researcher's goal was to examine the largest difference, all six MCPs tended to provide reasonable control of Type I errors. However, Alpha-Max and LSD along with the SNK and HSD tended to produce empirical α's most closely resembling the nominal α's. The Dunn and Scheffé tended to be overly conservative. However, as previously noted, this scenario is not likely to be important to most researchers. Most researchers would be interested in controlling the total Type I error.

The results when comparing the total Type I error were somewhat different. The HSD and Dunn tend to provide the best estimates with HSD results being slightly liberal and Dunn results being slightly conservative. The LSD and Alpha-Max clearly do not provide reasonable control for Type I error. Scheffé results are conservative as always and tend to become more so as the number of groups increase.

While the results do not provide a clear mandate for selection of one procedure in every situation, some generalizations can be made. The Alpha-Max procedure and the LSD procedure tend to provide similar results. The Scheffé procedure tends to be quite conservative in most situations. On the

average, the HSD and Dunn procedures tend to provide empirical α's closest to the nominal α's, but HSD tends to be slightly liberal while the Dunn tends to be slightly conservative.

Finally, the study does not provide support for using the Alpha-Max in place of some of the more traditional MCPs for post hoc multiple comparison procedures. The initial idea in the thinking for the Alpha-Max was that it could provide a more powerful a priori MCP. This still may be true and research should be conducted to determine if it is. Alpha-Max remains an alternative in the sense that it is still probably the easiest to implement. The jury remains out on its use as an a priori procedure. In addition, it's vaibility as an alternative when assumptions such as homogeneity of variance with unequal group sizes is to be assessed.

References

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Table 1. Observed Type I Error Proportion for First Type I Error, Multiple Comparison Procedure by Sample Size, K= 3, α= .05, 10 Replications of 1000 Each

Sample Size

Multiple Comparison Procedure	n= 5	n= 20	n= 35	n= 50	Total
LSD	.0564	.0496	.0527	.0508	.0524
Alpha-Max	.0564	.0496	.0527	.0508	.0524
Newman-Keuls	.0523	.0441	.0465	.0466	.0474
HSD	.0523	.0441	.0465	.0466	.0474
Dunn	.0470	.0405	.0432	.0425	.0433
Scheffé	.0465	.0363	.0404	.0397	.0407
Total	.0518	.0440	.0470	.0462	.0473

Table 2. ANOVA Summary Table, First Type I Error, Multiple Comparison Procedure by Sample Size, K= 3, α= .05, 10 Replications of 1000 Each

Source	df	MS*	F	p	η^2
Procedure	5	8.8592	15.41	.0001	.235
Sample size	3	6.4875	11.28	.0001	.103
Interaction	15	0.0488	0.08	1.000	.004
Error	216	0.5750			
Total	239				

* x 10^{-4}

Procedure Pairwise Differences:

	AMx	SNK	HSD	Dun	Sch
LSD		*	*	*	*
Amx		*	*	*	*
SNK					*
HSD					*
Dun					

Table 3. Observed Type I Error Proportion for First Type I Error, Multiple Comparison Procedure by Sample Size, K= 4, α= .05, 10 Replications of 1000 Each

Multiple Comparison Procedure	Sample Size n= 5	n= 20	n= 35	n= 50	Total
LSD	.0459	.0434	.0507	.0502	.0476
Alpha-Max	.0459	.0434	.0507	.0502	.0476
Newman-Keuls	.0390	.0352	.0448	.0419	.0402
HSD	.0390	.0352	.0448	.0419	.0402
Dunn	.0326	.0318	.0379	.0374	.0349
Scheffé	.0275	.0232	.0255	.0261	.0256
Total	.0383	.0354	.0424	.0413	.0393

Table 4. ANOVA Summary Table, First Type I Error, Multiple Comparison Procedure by Sample Size, K= 4, α= .05, 10 Replications of 1000 Each

Source	df	MS*	F	p	η^2
Procedure	5	27.6274	63.72	.0001	.545
Sample size	3	5.9949	13.83	.0001	.071
Interaction	15	0.2312	0.53	.9202	.014
Error	216	0.4336			
Total	239				

* x 10^{-4}

Procedure Pairwise Differences:

	AMx	SNK	HSD	Dun	Sch
LSD	*	*	*	*	
Amx	*	*	*	*	
SNK			*	*	
HSD			*	*	
Dun				*	

Table 5. Observed Type I Error Proportion for First Type I Error, Multiple Comparison Procedure by Sample Size, K= 5, α= .05, 10 Replications of 1000 Each

Multiple Comparison Procedure	Sample Size n= 5	n= 20	n= 35	n= 50	Total
LSD	.0514	.0513	.0471	.0498	.0499
Alpha-Max	.0514	.0513	.0471	.0498	.0499
Newman-Keuls	.0423	.0394	.0392	.0402	.0403
HSD	.0423	.0394	.0392	.0402	.0403
Dunn	.0352	.0355	.0334	.0350	.0347
Scheffé	.0220	.0180	.0181	.0179	.0190
Total	.0408	.0391	.0374	.0388	.0390

Table 6. ANOVA Summary Table, First Type I Error, Multiple Comparison Procedure by Sample Size, K= 5, α= .05, 10 Replications of 1000 Each

Source	df	MS*	F	p	η^2
Procedure	5	52.7835	163.17	.0001	.779
Sample size	3	1.1763	3.64	.0137	.010
Interaction	15	0.1001	0.31	.9942	.004
Error	216	0.3235			
Total	239				

* x 10^{-4}

Procedure Pairwise Differences:

	AMx	SNK	HSD	Dun	Sch
LSD	*	*	*	*	
AMx	*	*	*	*	
SNK			*	*	
HSD			*	*	
Dun				*	

Table 7. Observed Type I Error Proportion for First Type I Error, Multiple Comparison Procedure by Sample Size, K= 3, α= .01, 10 Replications of 1000 Each

Multiple Comparison Procedure	Sample Size n= 5	n= 20	n= 35	n= 50	Total
LSD	.0105	.0111	.0099	.0096	.0103
Alpha-Max	.0105	.0111	.0099	.0096	.0103
Newman-Keuls	.0096	.0099	.0091	.0085	.0093
HSD	.0096	.0099	.0091	.0085	.0093
Dunn	.0089	.0096	.0091	.0083	.0090
Scheffé	.0085	.0084	.0077	.0071	.0079
Total	.0096	.0100	.0091	.0086	.0093

Table 8. ANOVA Summary Table, First Type I Error, Multiple Comparison Procedure by Sample Size, K= 3, α= .01, 10 Replications of 1000 Each

Source	df	MS*	F	p	η^2
Procedure	5	0.3114	4.40	.0008	.089
Sample size	3	0.2187	3.09	.0279	.037
Interaction	15	0.0034	0.05	1.0000	.000
Error	216	0.0707			
Total	239				

* x 10^{-4}

Procedure Pairwise Differences:

	AMx	SNK	HSD	Dun	Sch
LSD					*
AMx					*
SNK					
HSD					
Dun					

Table 9. Observed Type I Error Proportion for First Type I Error, Multiple Comparison Procedure by Sample Size, K= 4, α= .01, 10 Replications of 1000 Each

Multiple Comparison Procedure	Sample Size n= 5	n= 20	n= 35	n= 50	Total
LSD	.0097	.0093	.0108	.0112	.0103
Alpha-Max	.0097	.0093	.0108	.0112	.0103
Newman-Keuls	.0069	.0072	.0091	.0093	.0081
HSD	.0069	.0072	.0091	.0093	.0081
Dunn	.0060	.0067	.0079	.0080	.0072
Scheffé	.0045	.0044	.0041	.0050	.0045
Total	.0073	.0074	.0086	.0090	.0081

Table 10. ANOVA Summary Table, First Type I Error, Multiple Comparison Procedure by Sample Size, K= 4, α= .01, 10 Replications of 1000 Each

Source	df	MS*	F	p	η^2
Procedure	5	1.8482	41.56	.0001	.449
Sample size	3	0.4639	10.43	.0001	.068
Interaction	15	0.0234	0.53	.9246	.017
Error	216	0.0445			
Total	239				

* x 10^{-4}

Procedure Pairwise Differences:

	AMx	SNK	HSD	Dun	Sch
LSD	*	*	*	*	
AMx	*	*	*	*	
SNK				*	
HSD				*	
Dun				*	

Table 11. Observed Type I Error Proportion for First Type I Error, Multiple Comparison Procedure by Sample Size, K= 5, α= .01, 10 Replications of 1000 Each

Multiple Comparison Procedure	Sample Size n= 5	n= 20	n= 35	n= 50	Total
LSD	.0100	.0111	.0101	.0102	.0104
Alpha-Max	.0100	.0111	.0101	.0102	.0104
Newman-Keuls	.0080	.0082	.0073	.0076	.0078
HSD	.0080	.0082	.0073	.0076	.0078
Dunn	.0068	.0078	.0061	.0066	.0068
Scheffé	.0037	.0031	.0025	.0029	.0031
Total	.0078	.0083	.0072	.0075	.0077

Table 12. ANOVA Summary Table, First Type I Error, Multiple Comparison Procedure by Sample Size, K= 5, α= .01, 10 Replications of 1000 Each

Source	df	MS*	F	p	η^2
Procedure	5	2.9155	37.58	.0001	.458
Sample size	3	0.1112	1.43	.2342	.010
Interaction	15	0.0097	0.13	1.0000	.005
Error	216	0.0776			
Total	239				

* x 10^{-4}

Procedure Pairwise Differences:

	AMx	SNK	HSD	Dun	Sch
LSD	*	*	*	*	
AMx	*	*	*	*	
SNK				*	
HSD				*	
Dun				*	

Table 13. Observed Type I Error Proportion for Total Type I Errors, Multiple Comparison Procedure by Sample Size, K= 3, α= .05, 10 Replications of 1000 Each

Multiple Comparison Procedure	Sample Size n= 5	n= 20	n= 35	n= 50	Total
LSD	.0923	.0755	.0785	.0789	.0813
Alpha-Max	.0823	.0681	.0712	.0709	.0734
Newman-Keuls	.0846	.0652	.0671	.0709	.0720
HSD	.0659	.0522	.0539	.0571	.0573
Dunn	.0584	.0471	.0496	.0503	.0514
Scheffé	.0574	.0420	.0455	.0464	.0478
Total	.0736	.0584	.0610	.0624	.0638

Table 14. ANOVA Summary Table, Total Type I Errors, Multiple Comparison Procedure by Sample Size, K= 3, α= .05, 10 Replications of 1000 Each

Source	df	MS*	F	p	η^2
Procedure	5	73.3315	71.18	.0001	.544
Sample size	3	27.2663	26.46	.0001	.121
Interaction	15	0.2148	0.21	.9994	.005
Error	216	1.0303			
Total	239				

* x 10^{-4}

Procedure Pairwise Differences:

	AMx	SNK	HSD	Dun	Sch
LSD	*	*	*	*	*
AMx			*	*	*
SNK			*	*	*
HSD					*
Dun					

Table 15. Observed Type I Error Proportion for Total Type I Errors, Multiple Comparison Procedure by Sample Size, K= 4, α= .05, 10 Replications of 1000 Each

	Sample Size				
Multiple Comparison Procedure	n= 5	n= 20	n= 35	n= 50	Total
LSD	.1116	.0988	.1117	.1122	.1086
Alpha-Max	.0970	.0868	.0974	.0987	.0950
Newman-Keuls	.0733	.0611	.0725	.0703	.0693
HSD	.0532	.0445	.0558	.0526	.0515
Dunn	.0437	.0397	.0450	.0450	.0434
Scheffé	.0348	.0275	.0291	.0294	.0302
Total	.0689	.0597	.0686	.0680	.0663

Table 16. ANOVA Summary Table, Total Type I Errors, Multiple Comparison Procedure by Sample Size, K= 4, α= .05, 10 Replications of 1000 Each

Source	df	MS*	F	p	η^2
Procedure	5	373.3313	235.46	.0001	.829
Sample size	3	11.6544	7.35	.0001	.016
Interaction	15	0.5595	0.35	.9883	.004
Error	216	1.5855			
Total	239				

* x 10^{-4}

Procedure Pairwise Differences:

	AMx	SNK	HSD	Dun	Sch
LSD	*	*	*	*	*
AMx		*	*	*	*
SNK			*	*	*
HSD				*	*
Dun					*

Table 17. Observed Type I Error Proportion for Total Type I Errors, Multiple Comparison Procedure by Sample Size, K= 5, α= .05, 10 Replications of 1000 Each

Multiple Comparison Procedure	Sample Size n= 5	n= 20	n= 35	n= 50	Total
LSD	.1766	.1655	.1497	.1590	.1627
Alpha-Max	.1379	.1340	.1222	.1299	.1310
Newman-Keuls	.0839	.0785	.0754	.0782	.0790
HSD	.0609	.0567	.0550	.0583	.0577
Dunn	.0462	.0471	.0447	.0482	.0466
Scheffé	.0275	.0227	.0220	.0220	.0236
Total	.0888	.0841	.0782	.0826	.0834

Table 18. ANOVA Summary Table, Total Type I Errors, Multiple Comparison Procedure by Sample Size, K= 5, α= .05, 10 Replications of 1000 Each

Source	df	MS*	F	p	η^2
Procedure	5	1133.8206	544.26	.0001	.917
Sample size	3	11.6028	5.57	.0011	.006
Interaction	15	1.7025	0.82	.6579	.004
Error	216	2.0832			
Total	239				

* x 10^{-4}

Procedure Pairwise Differences:

	AMx	SNK	HSD	Dun	Sch
LSD	*	*	*	*	*
AMx		*	*	*	*
SNK			*	*	*
HSD				*	*
Dun					*

Table 19. Observed Type I Error Proportion for Total Type I Errors, Multiple Comparison Procedure by Sample Size, K= 3, α= .01, 10 Replications of 1000 Each

Multiple Comparison Procedure	Sample Size n= 5	n= 20	n= 35	n= 50	Total
LSD	.0155	.0138	.0127	.0130	.0138
Alpha-Max	.0148	.0131	.0119	.0119	.0129
Newman-Keuls	.0139	.0117	.0116	.0112	.0121
HSD	.0111	.0106	.0105	.0097	.0105
Dunn	.0102	.0103	.0104	.0093	.0101
Scheffé	.0097	.0088	.0088	.0079	.0088
Total	.0125	.0114	.0110	.0105	.0114

Table 20. ANOVA Summary Table, Total Type I Errors, Multiple Comparison Procedure by Sample Size, K= 3, α= .01, 10 Replications of 1000 Each

Source	df	MS*	F	p	η^2
Procedure	5	0.1579	12.85	.0001	.216
Sample size	3	0.0502	4.09	.0076	.041
Interaction	15	0.0035	0.28	.9965	.014
Error	216	0.0123			
Total	239				

* x 10^{-4}

Procedure Pairwise Differences:

	AMx	SNK	HSD	Dun	Sch
LSD			*	*	*
AMx			*	*	*
SNK					*
HSD					
Dun					

Table 21. Observed Type I Error Proportion for Total Type I Errors, Multiple Comparison Procedure by Sample Size, K= 4, α= .01, 10 Replications of 1000 Each

Multiple Comparison Procedure	Sample Size n= 5	n= 20	n= 35	n= 50	Total
LSD	.0202	.0163	.0204	.0190	.0190
Alpha-Max	.0177	.0147	.0194	.0176	.0174
Newman-Keuls	.0103	.0089	.0143	.0118	.0113
HSD	.0085	.0083	.0116	.0106	.0098
Dunn	.0070	.0074	.0097	.0091	.0083
Scheffé	.0050	.0048	.0045	.0054	.0049
Total	.0115	.0101	.0133	.0123	.0118

Table 22. ANOVA Summary Table, Total Type I Errors, Multiple Comparison Procedure by Sample Size, K= 4, α= .01, 10 Replications of 1000 Each

Source	df	MS*	F	p	η^2
Procedure	5	11.6977	93.20	.0001	.645
Sample size	3	1.1253	8.97	.0001	.037
Interaction	15	0.1179	0.94	.5213	.019
Error	216	0.1255			
Total	239				

* x 10^{-4}

Procedure Pairwise Differences:

	AMx	SNK	HSD	Dun	Sch
LSD		*	*	*	*
AMx		*	*	*	*
SNK				*	*
HSD					*
Dun					*

Table 23. Observed Type I Error Proportion for Total Type I Errors, Multiple Comparison Procedure by Sample Size, K= 5, α= .01, 10 Replications of 1000 Each

Multiple Comparison Procedure	Sample Size n= 5	n= 20	n= 35	n= 50	Total
LSD	.0285	.0266	.0257	.0235	.0261
Alpha-Max	.0243	.0231	.0214	.0206	.0224
Newman-Keuls	.0152	.0122	.0117	.0114	.0126
HSD	.0109	.0108	.0096	.0098	.0103
Dunn	.0091	.0098	.0076	.0077	.0086
Scheffé	.0045	.0038	.0028	.0031	.0036
Total	.0154	.0144	.0131	.0127	.0139

Table 24. ANOVA Summary Table, Total Type I Errors, Multiple Comparison Procedure by Sample Size, K= 5, α= .01, 10 Replications of 1000 Each

Source	df	MS*	F	p	η^2
Procedure	5	29.6115	75.63	.0001	.626
Sample size	3	0.9204	2.35	.0733	.012
Interaction	15	0.0625	0.16	.9999	.004
Error	216	0.3915			
Total	239				

* x 10^{-4}

Procedure Pairwise Differences:

	AMx	SNK	HSD	Dun	Sch
LSD		*	*	*	*
AMx		*	*	*	*
SNK				*	*
HSD					*
Dun					*

Table 25. Observed Type I Error Proportion for Total Type I Errors, Total for Each Alpha Level

	α= .05			α= .01		
Multiple Comparison Procedure	K= 3	K= 4	K= 5	K= 3	K= 4	K= 5
LSD	.0813	.1086	.1627	.0138	.0190	.0261
Alpha-Max	.0734	.0950	.1310	.0129	.0174	.0224
Newman-Keuls	.0720	.0693	.0790	.0121	.0113	.0126
HSD	.0573	.0515	.0577	.0105	.0098	.0103
Dunn	.0514	.0434	.0466	.0101	.0083	.0086
Scheffé	.0478	.0302	.0236	.0088	.0049	.0036